Recent advances in lifted inference @ Leuven

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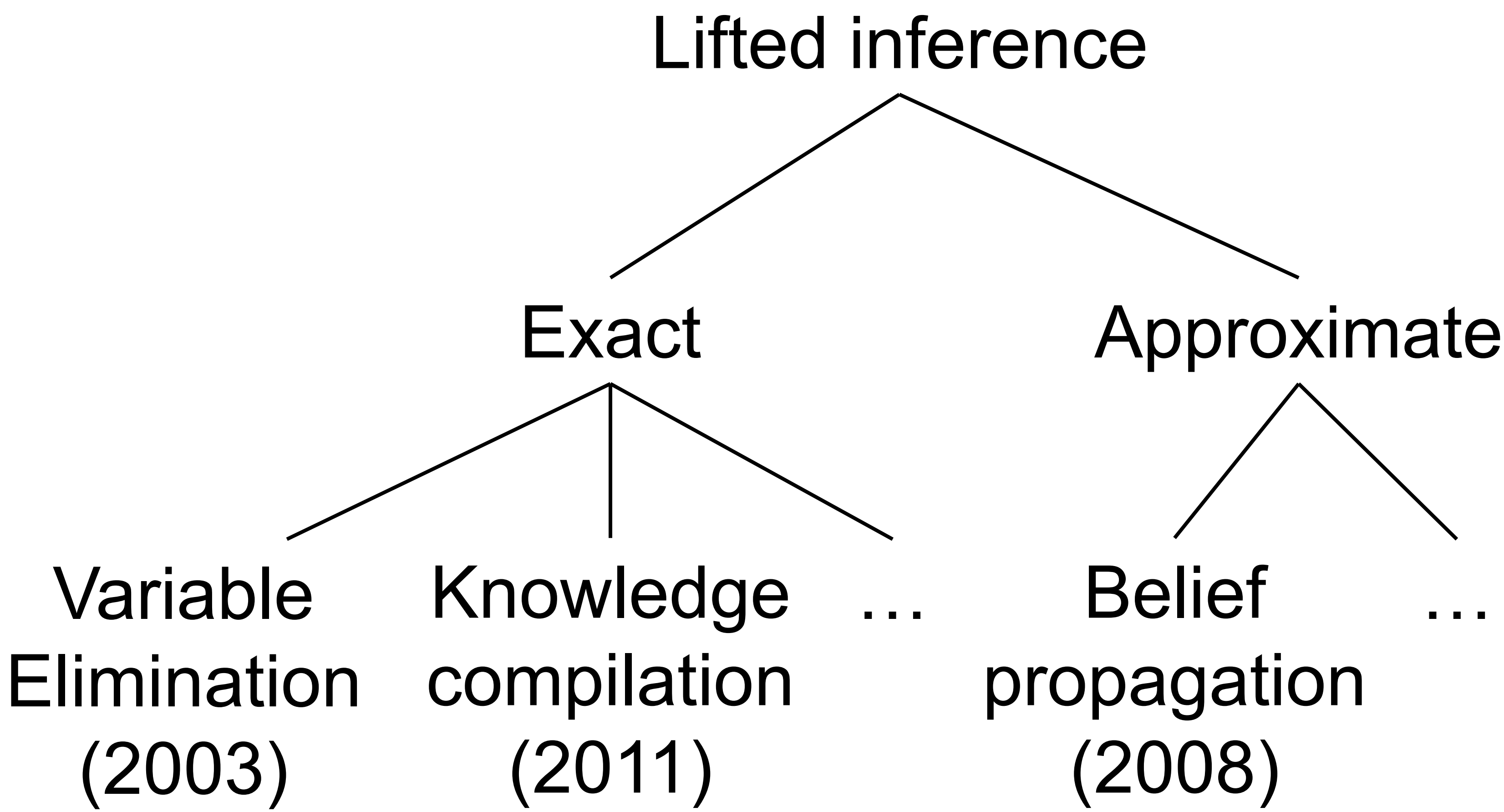
SML, April 19, 2012

Outline

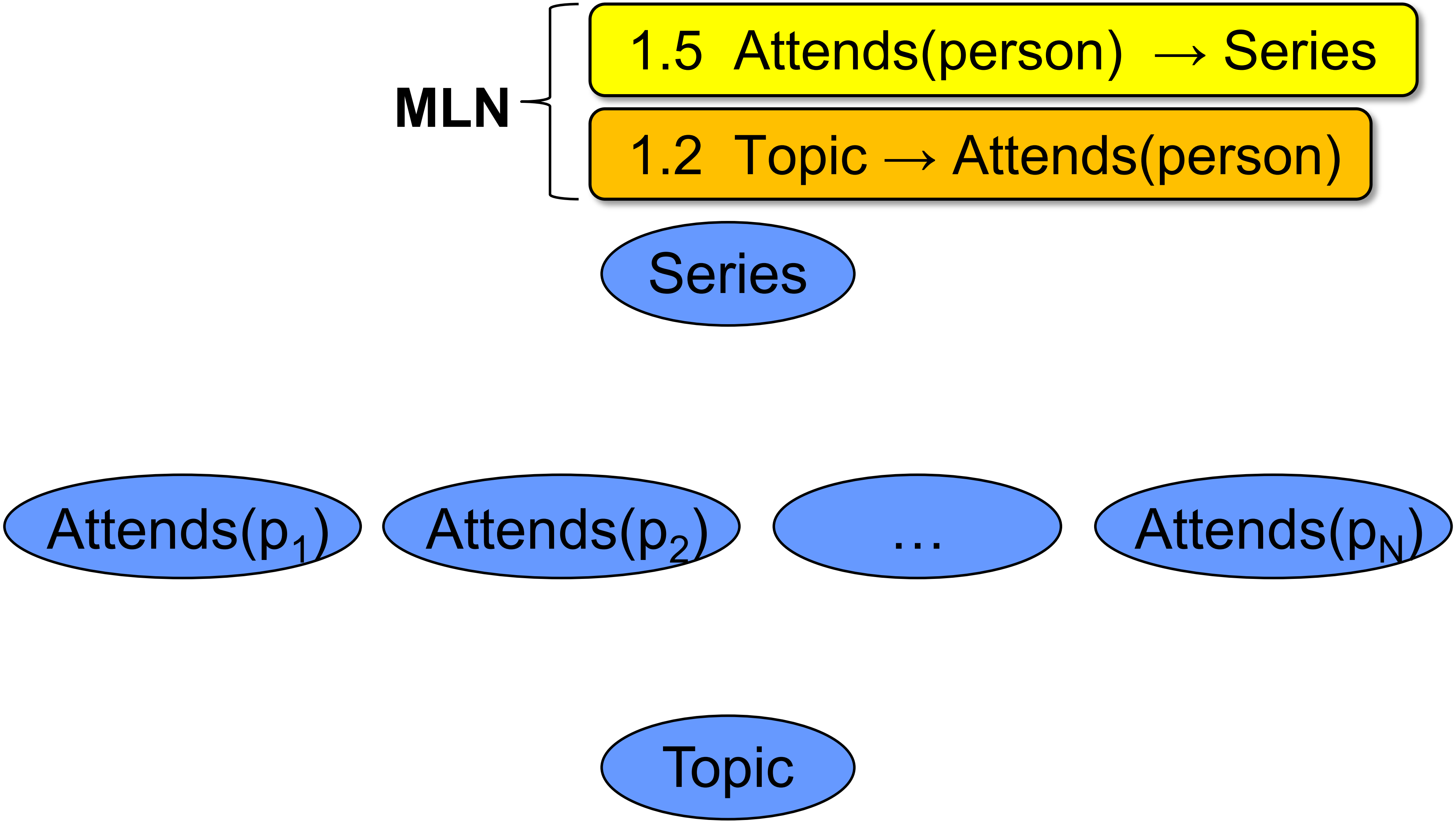
- Introduction to lifted inference
- Four contributions
  - Arbitrary constraints
  - Completeness results
  - Conditioning
  - An approximate method



and many more !

MLN

- 1.5 Attends(person) → Series
- 1.2 Topic → Attends(person)


MLN
1.5 Attends(person) → Series
1.2 Topic → Attends(person)
Series
Attends(p1)
Attends(p2)
…
Attends(pN)
Topic

MLN

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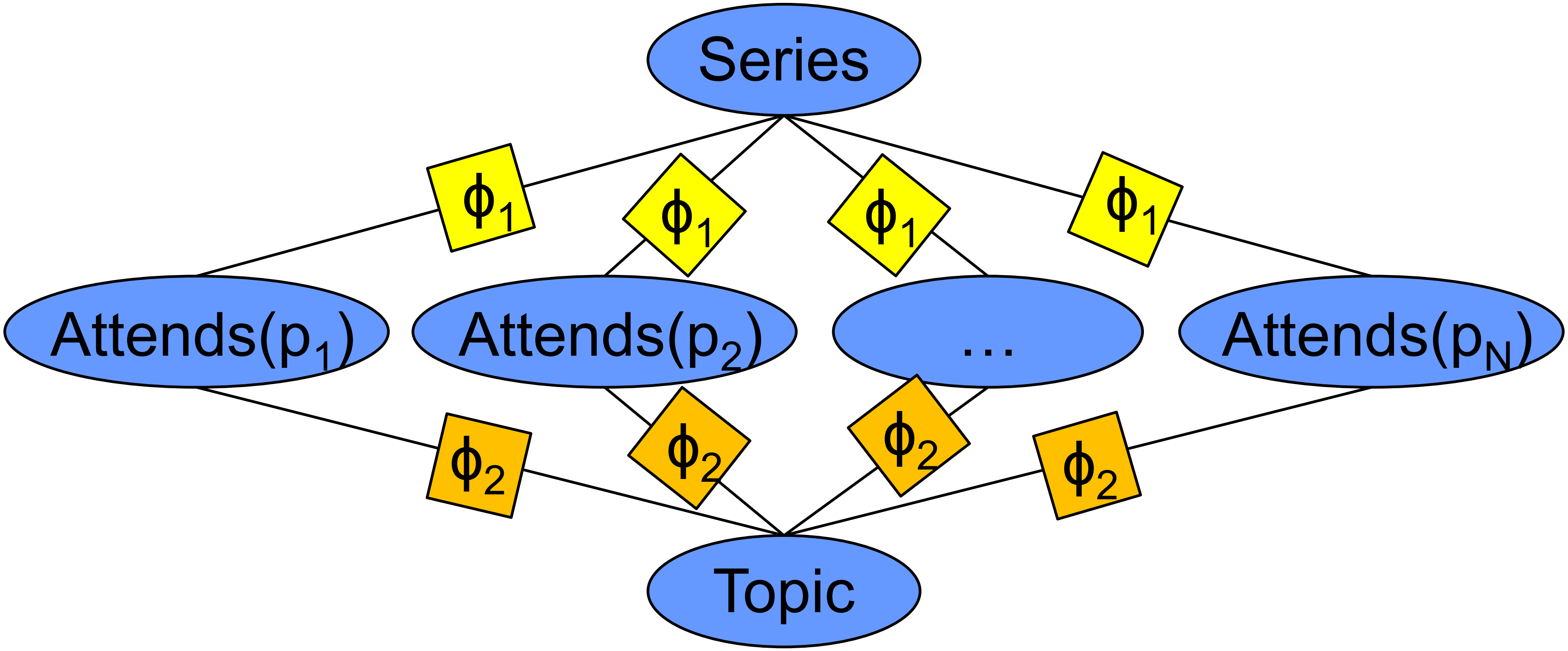

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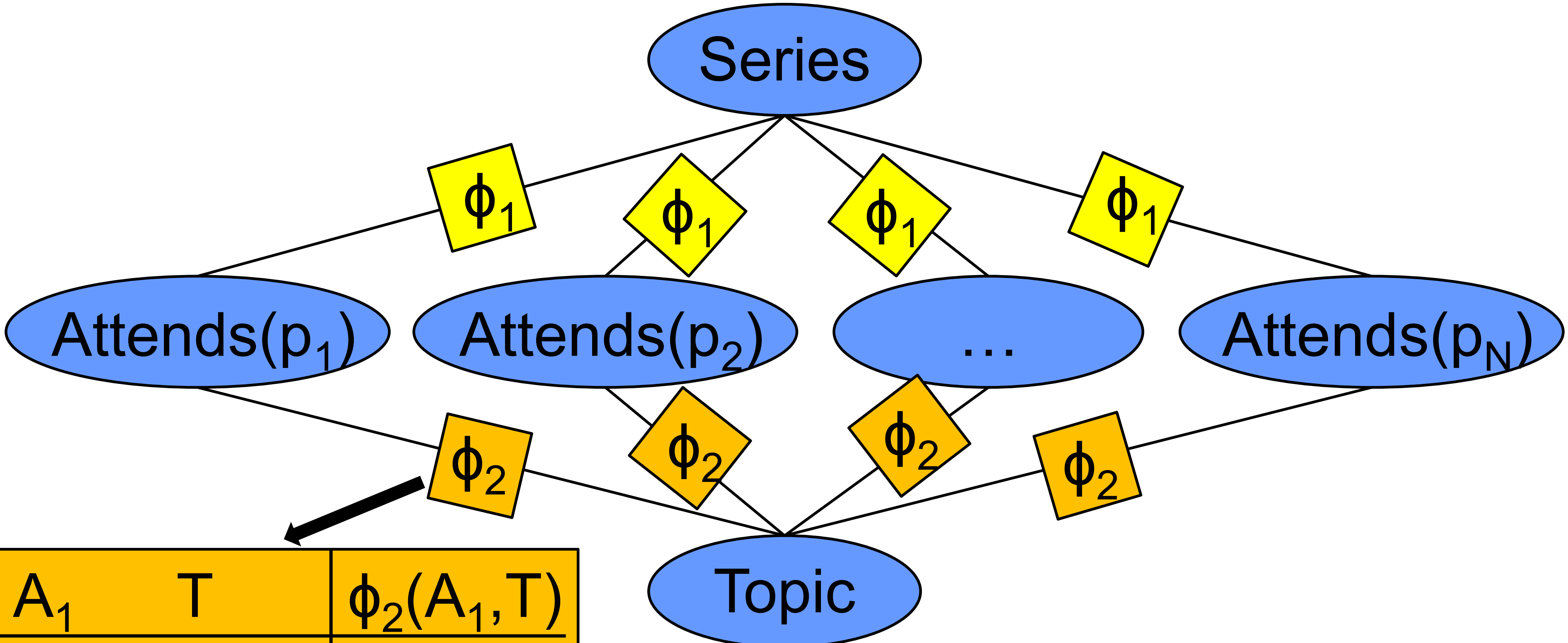

A_1	T	$\phi_2(A_1,T)$
true	true	3.3
true	false	3.3
false	true	1.0
false	false	3.3

MLN

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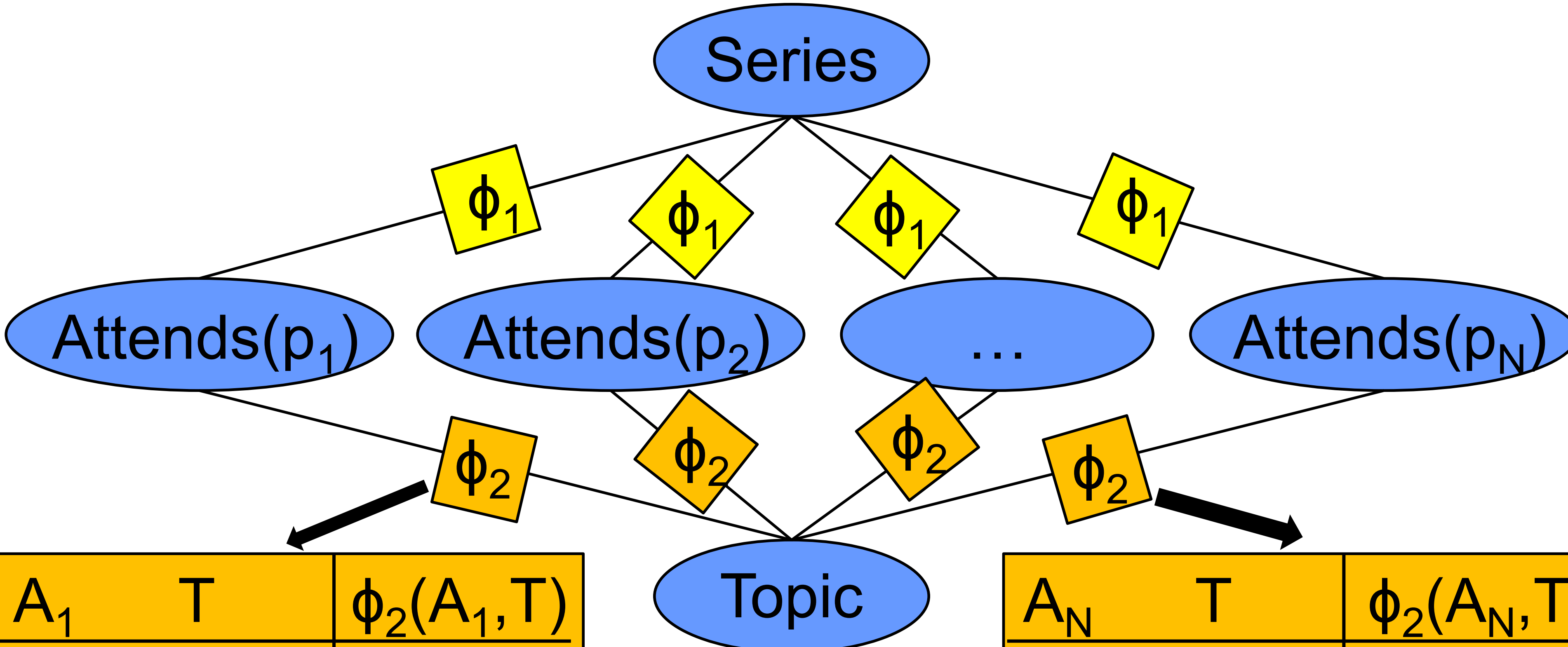
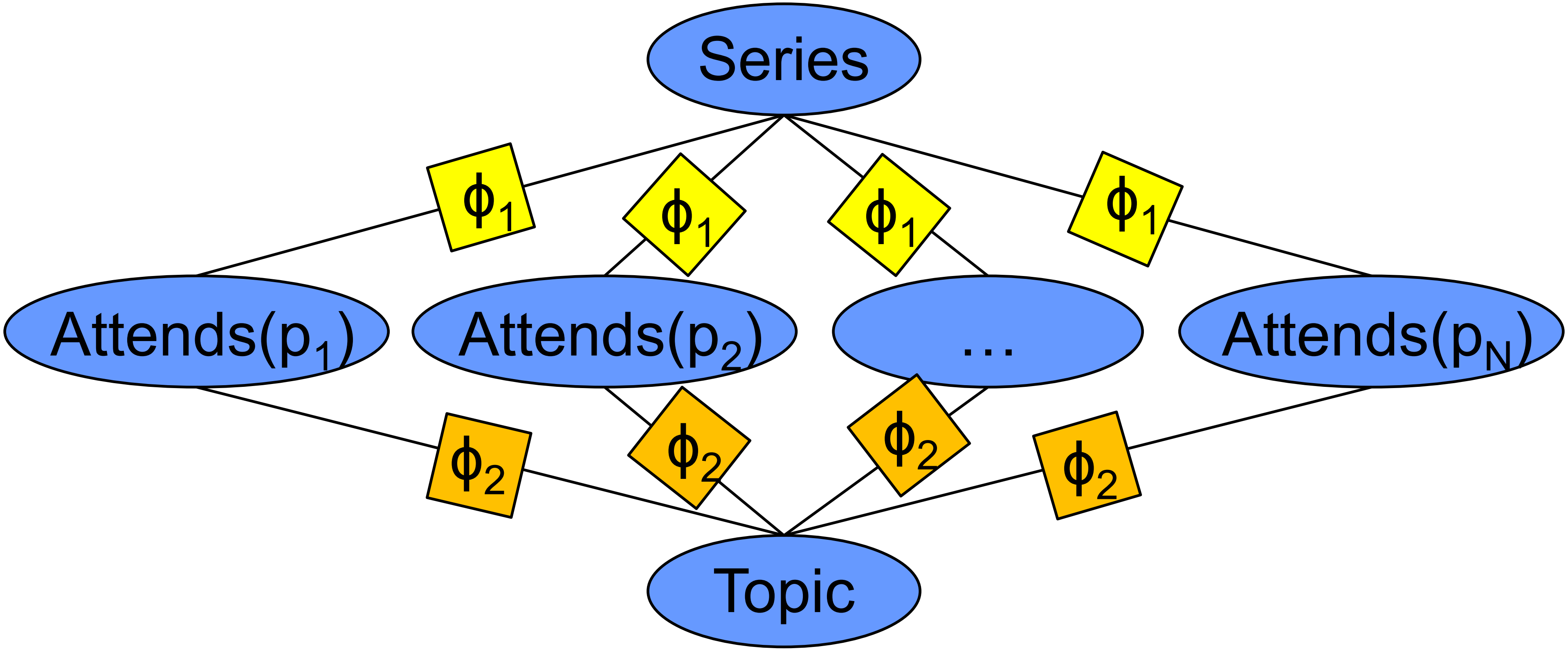
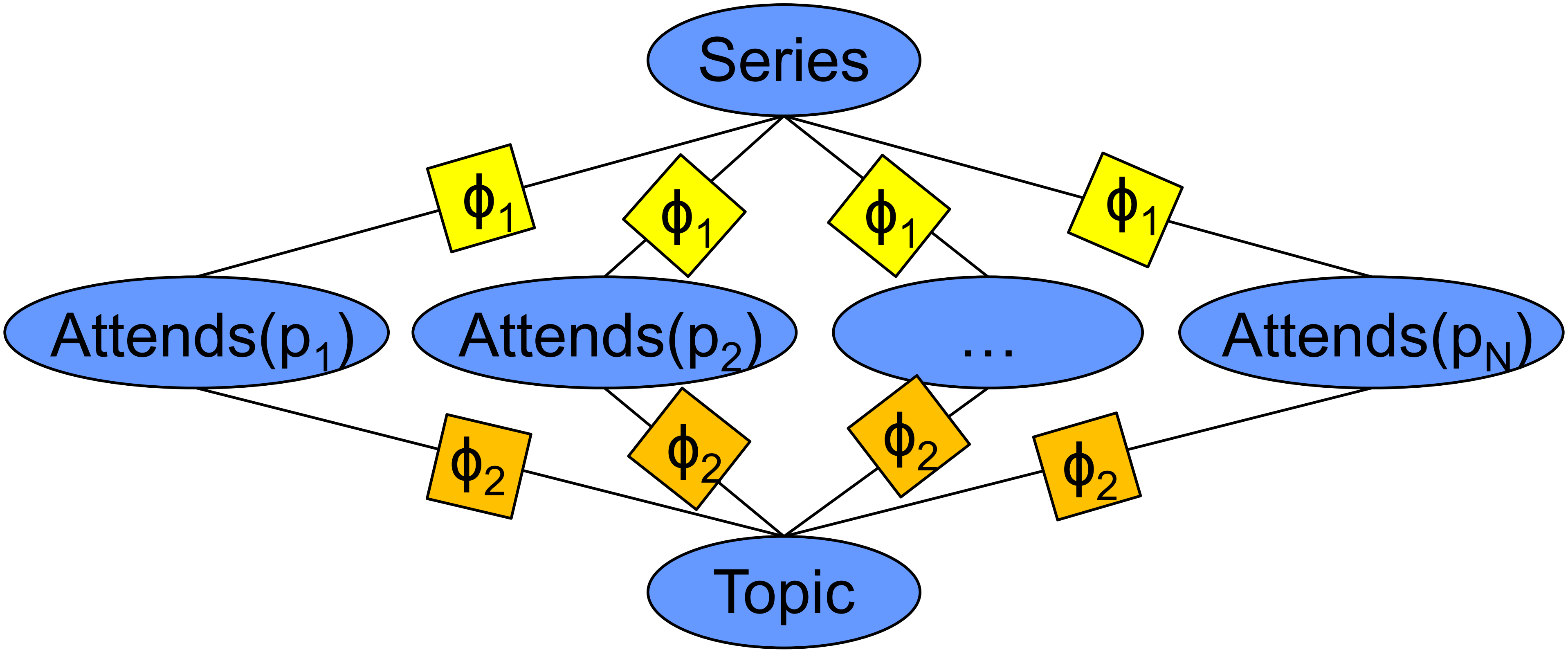


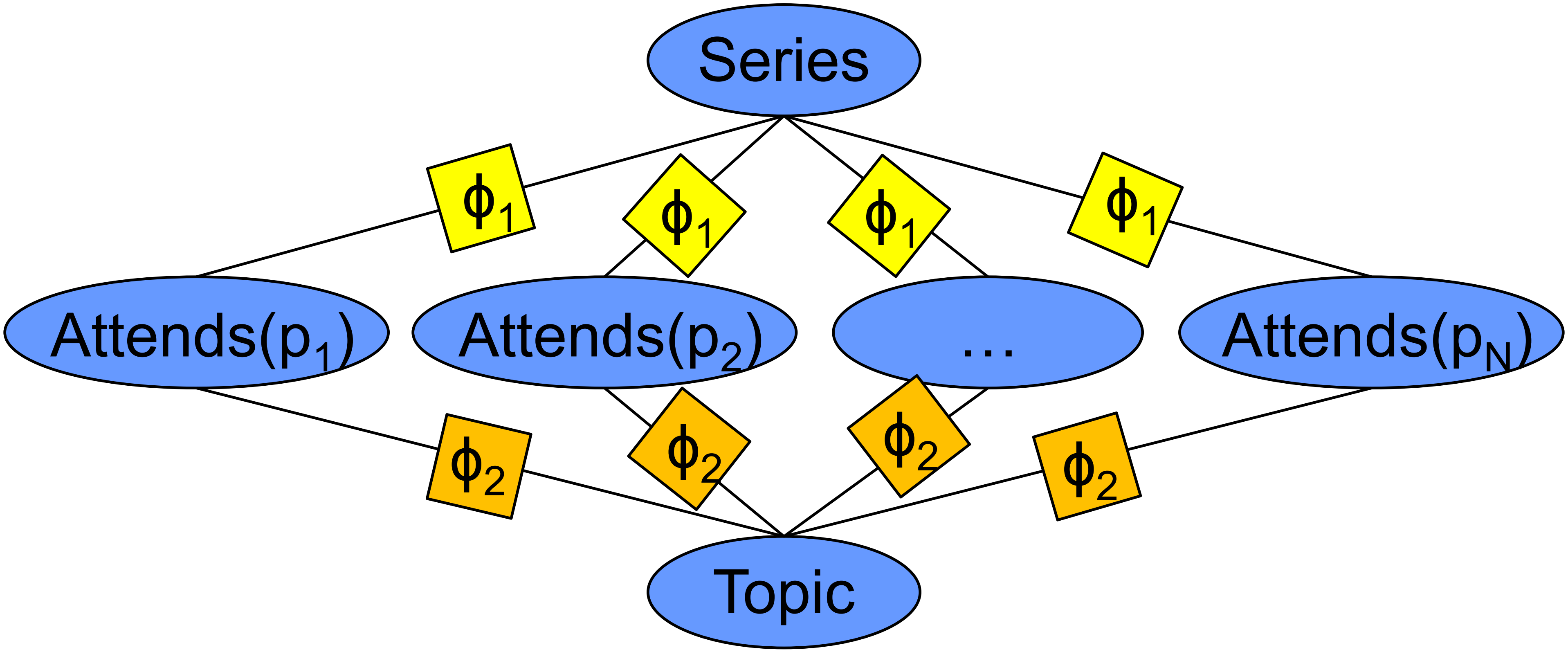
A_1	T	$\phi_2(A_1,T)$
true	true	3.3
true	false	3.3
false	true	1.0
false	false	3.3

A_N	T	$\phi_2(A_N,T)$
true	true	3.3
true	false	3.3
false	true	1.0
false	false	3.3


Series
ϕ_1
ϕ_1
ϕ_1
ϕ_1
Attends(p_1)
Attends(p_2)
...
Attends(p_N)
ϕ_2
ϕ_2
ϕ_2
ϕ_2
Topic

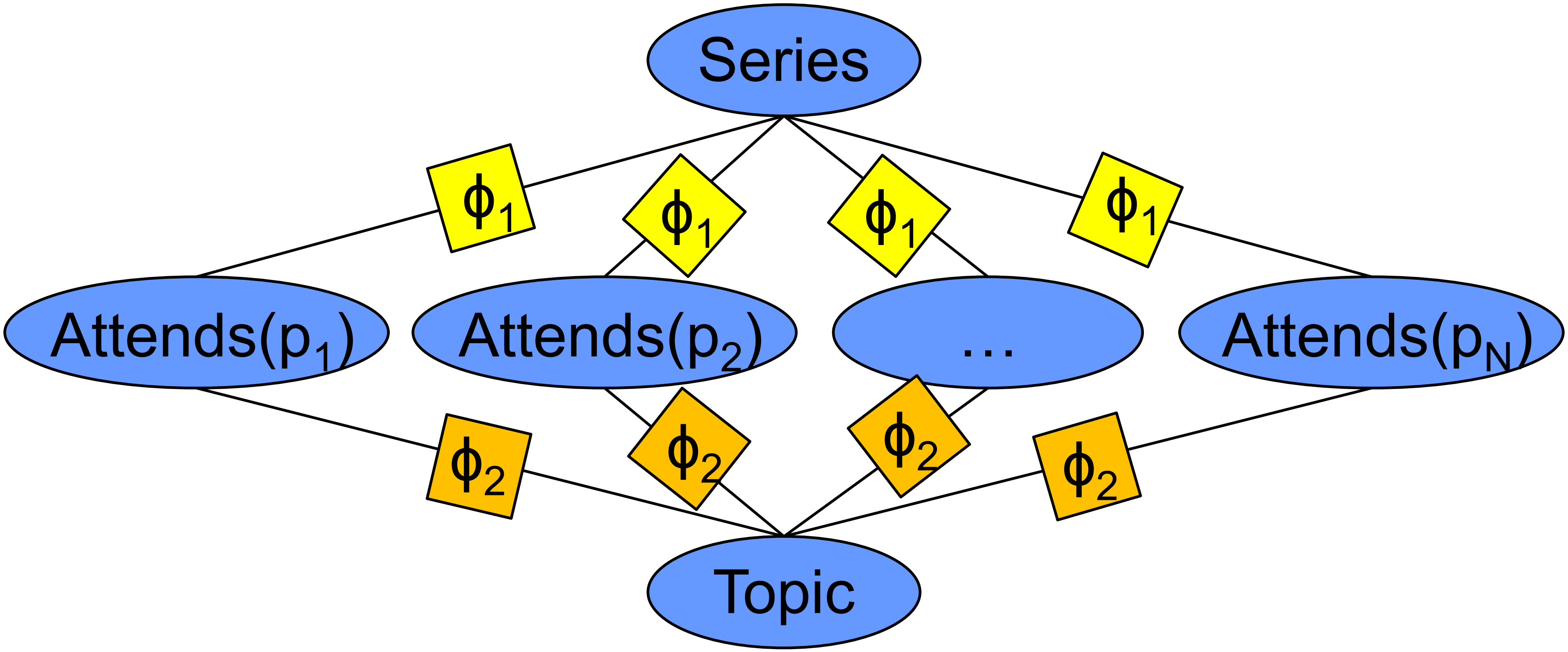


$$P(S, A_1, \ldots, A_N, T) = \frac{1}{Z} \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i)$$



$$P(S) = \frac{1}{Z} \sum_{T} \sum_{A_1} \ldots \sum_{A_N} \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i)$$

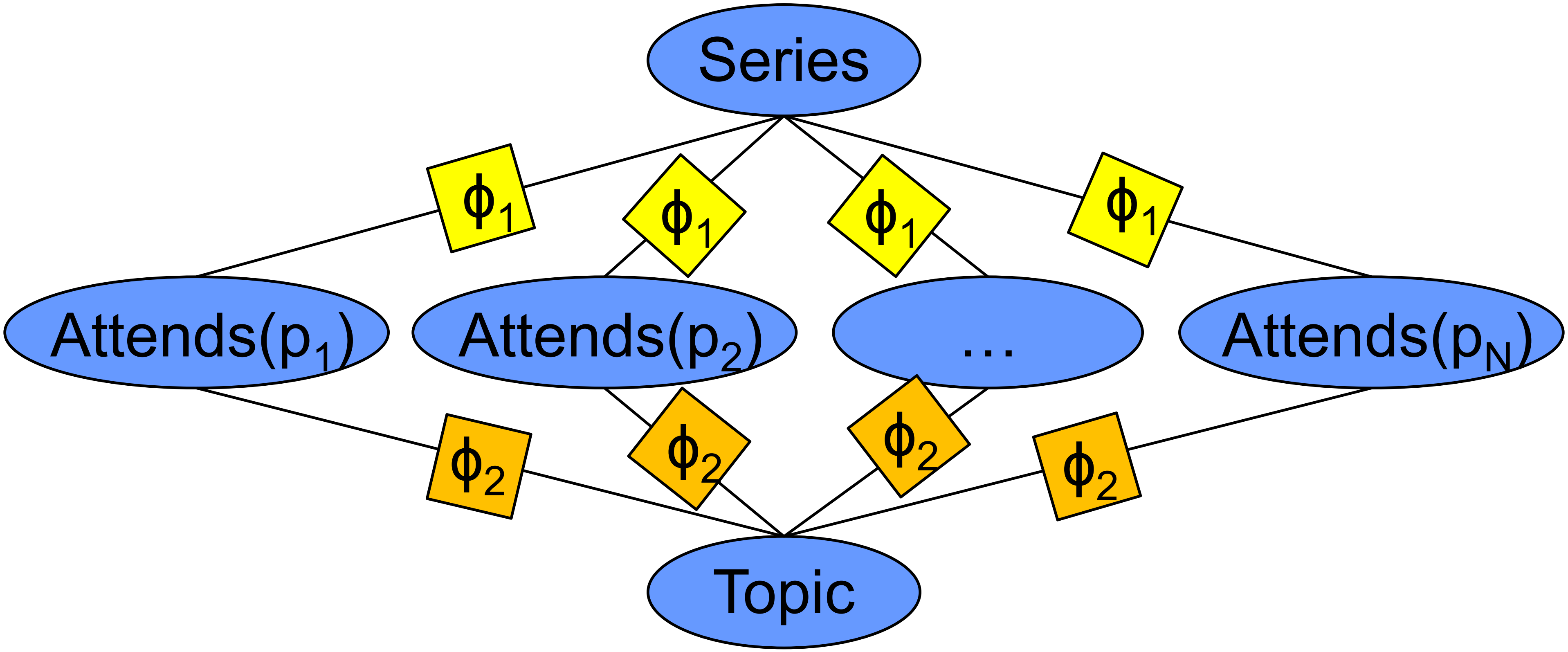
will it become a series ?



$$P(S) = \frac{1}{Z} \underbrace{\sum_{T} \sum_{A_1} \dots \sum_{A_N}}_{2^{(N+1)} \text{ terms}} \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i)$$

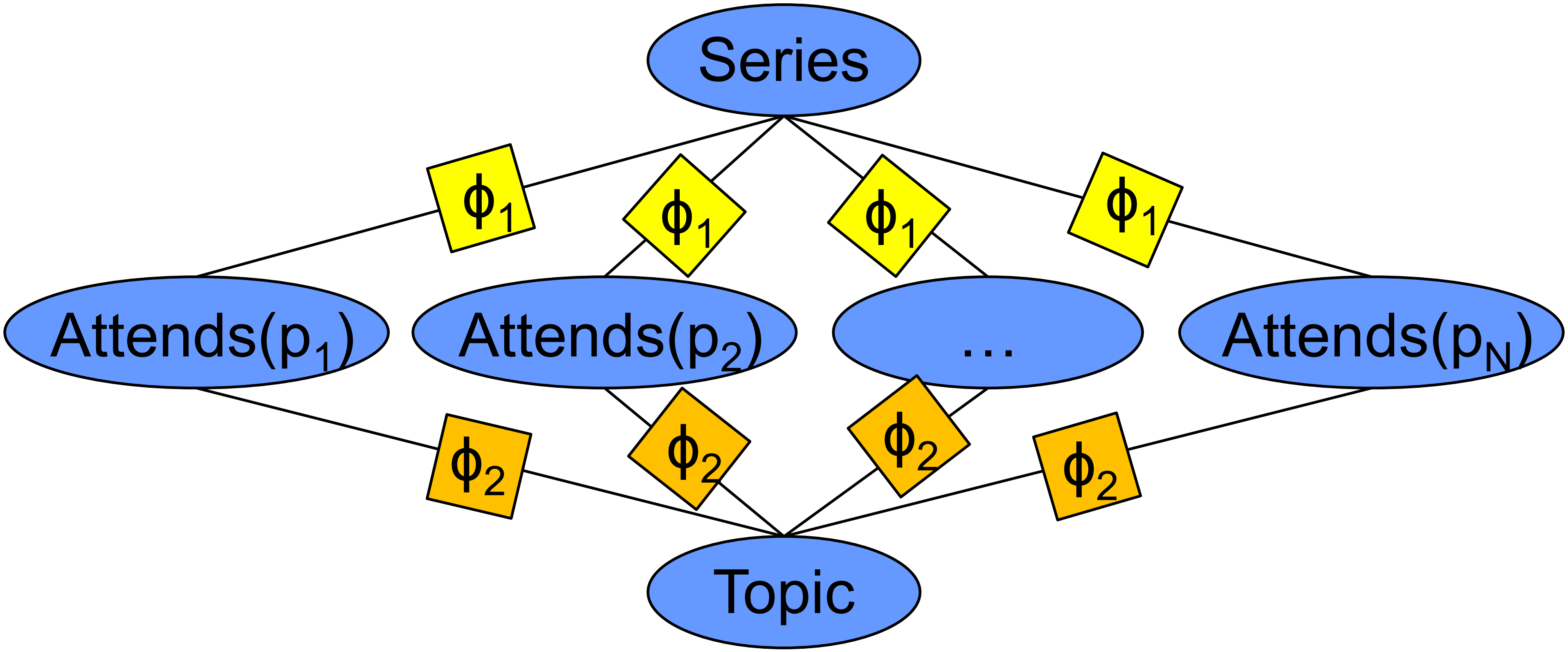
will it become a series ?

$2^{(N+1)}$ terms



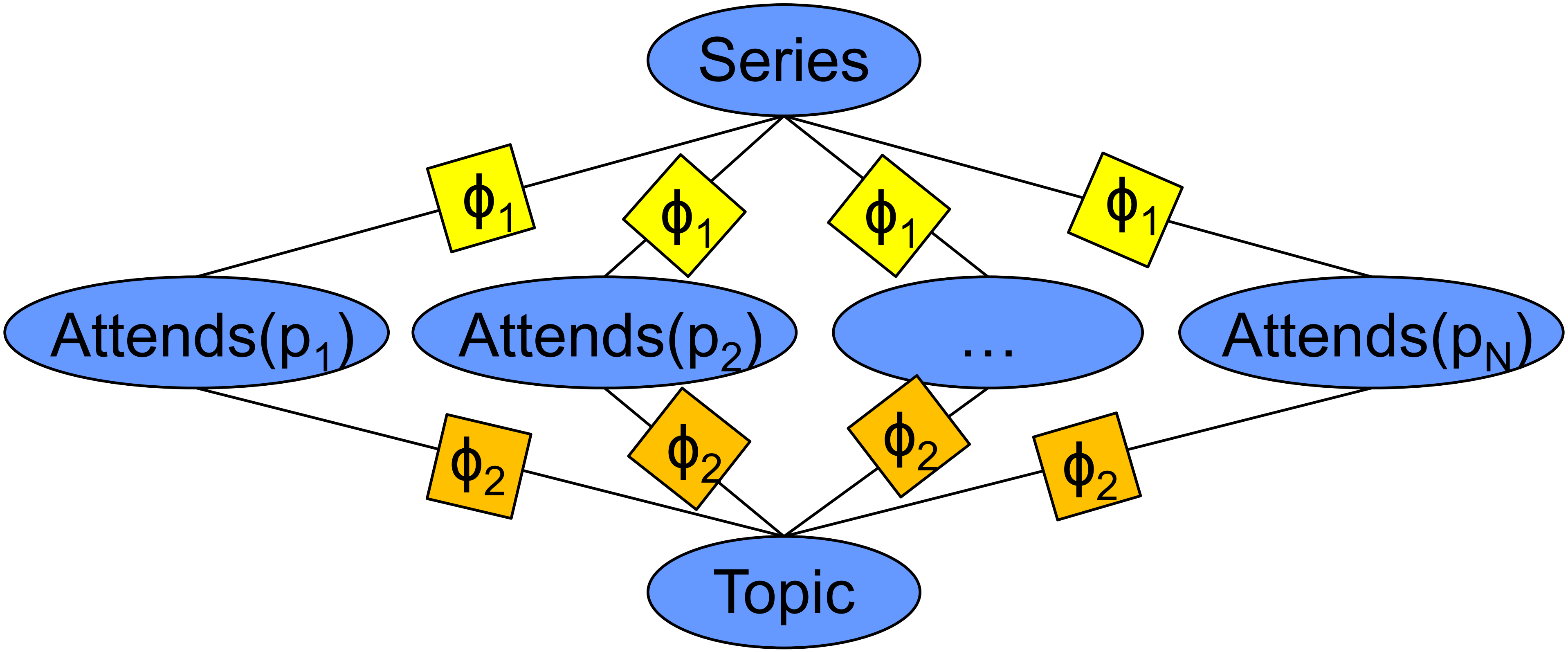
$$\sum_{T}\sum_{A_1}\cdots\sum_{A_N}\prod_{i=1}^{N}\phi_1(A_i, S)\prod_{i=1}^{N}\phi_2(T, A_i)$$

$2^{(N+1)}$ terms



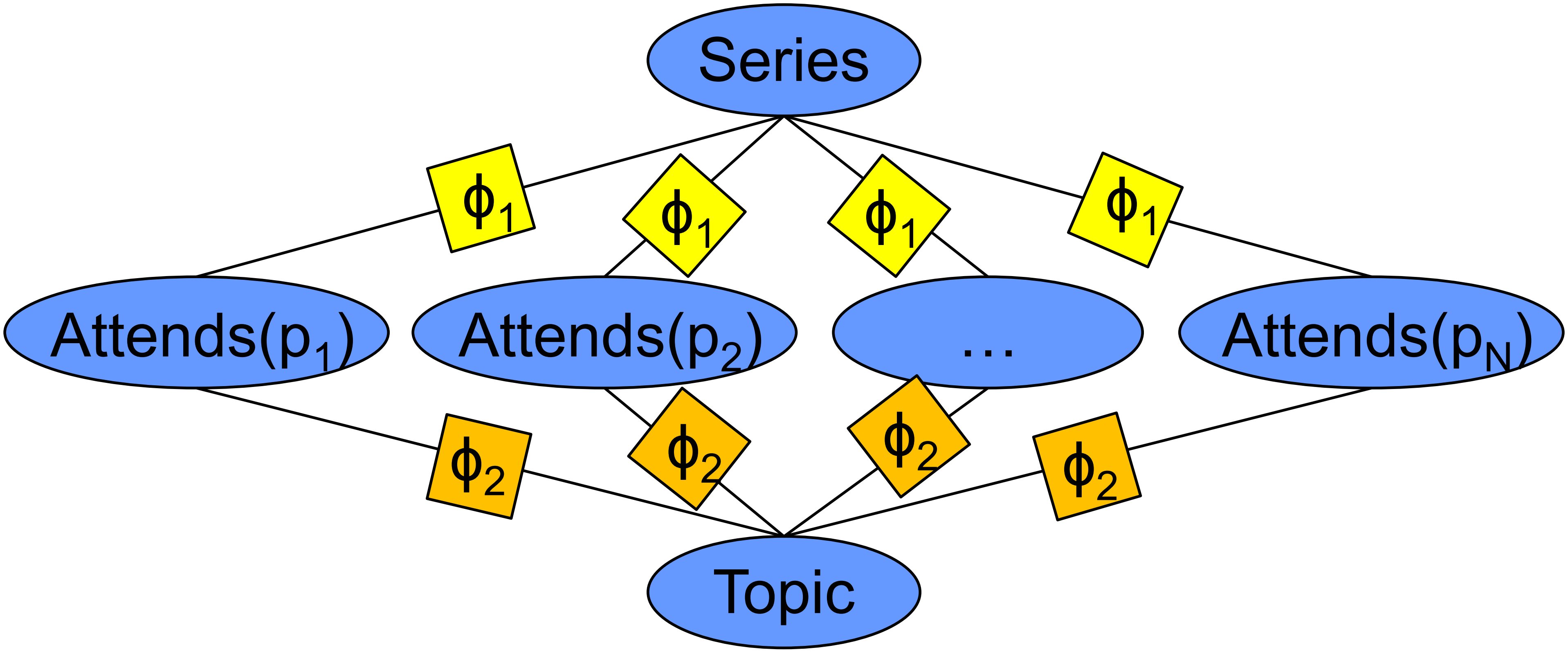
$$\sum_{T}\left(\sum_{A_1}\phi_1(A_1,S)\phi_2(T,A_1)\right)\cdots\left(\sum_{A_N}\phi_1(A_N,S)\phi_2(T,A_N)\right)$$

1 for every person



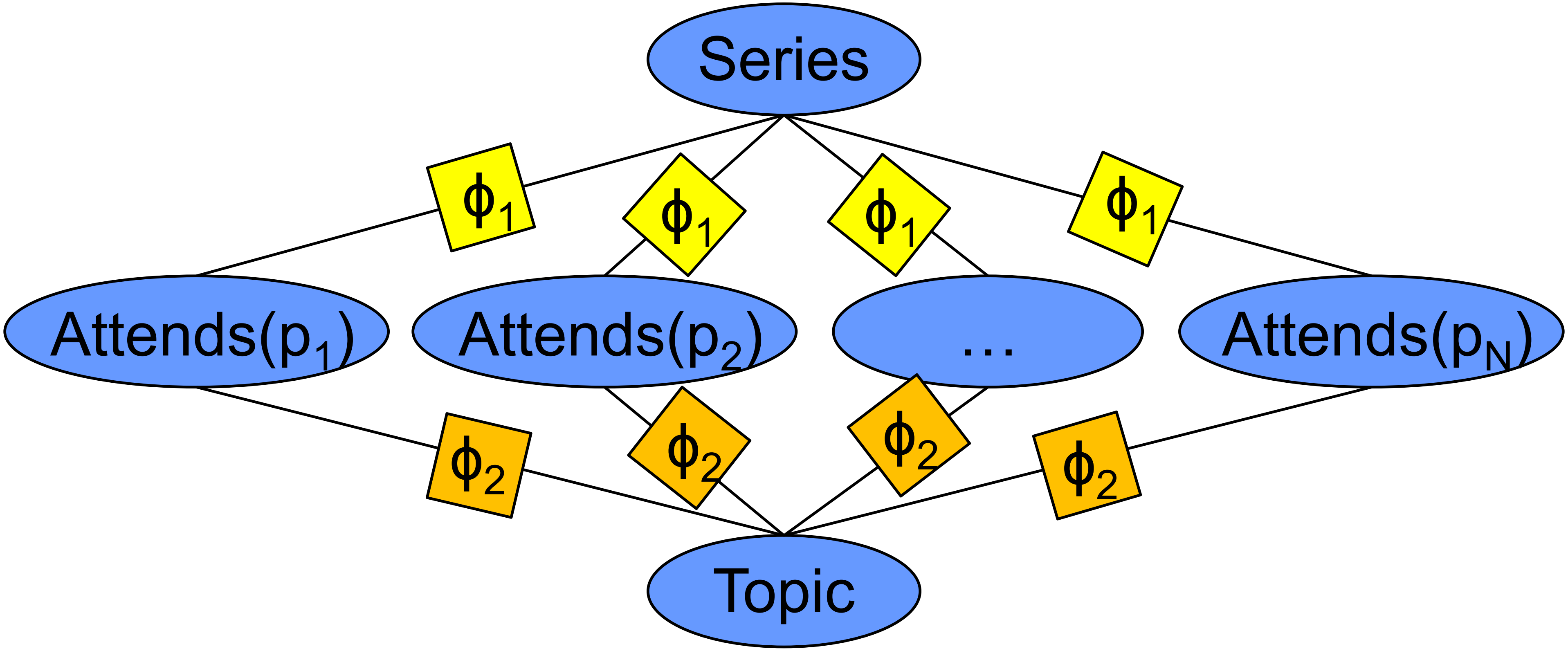
$$\sum_{T}\left(\sum_{A_1}\underbrace{\phi_1(A_1,S)\,\phi_2(T,A_1)}\right)\cdots\left(\sum_{A_N}\underbrace{\phi_1(A_N,S)\,\phi_2(T,A_N)}\right)$$

N times the same product !



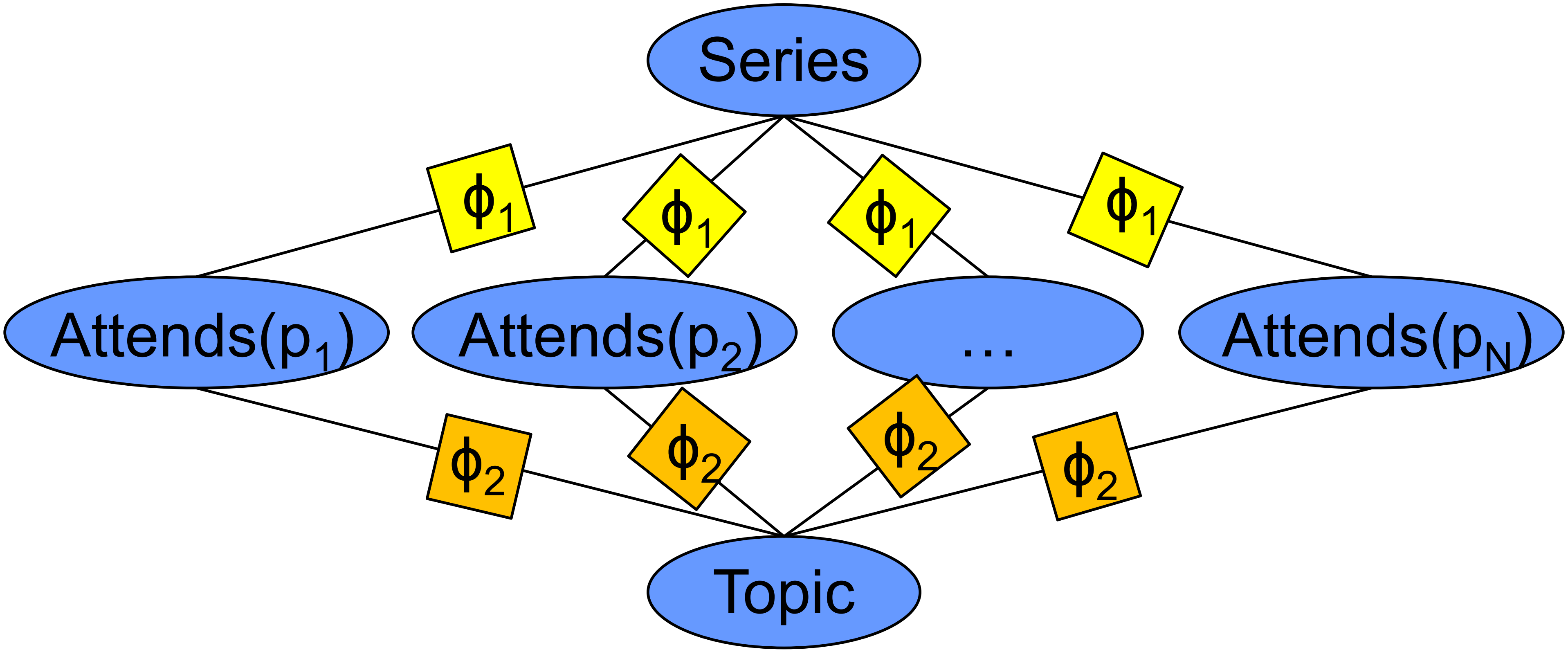
$$\sum_{T}\left(\sum_{A_1}\phi_1(A_1,S)\phi_2(T,A_1)\right)\cdots\left(\sum_{A_N}\phi_1(A_N,S)\phi_2(T,A_N)\right)$$

N times the same sum !



lifted: $$\sum_{T}\left(\sum_{A}\phi_1(A,S)\phi_2(T,A)\right)^N$$

compute only once !



lifted: $$\sum_T \left(\sum_A \phi_1(A, S)\phi_2(T, A) \right)^N$$

"lifted sum-out"

"lifted multiplication"

Lifted Variable Elimination

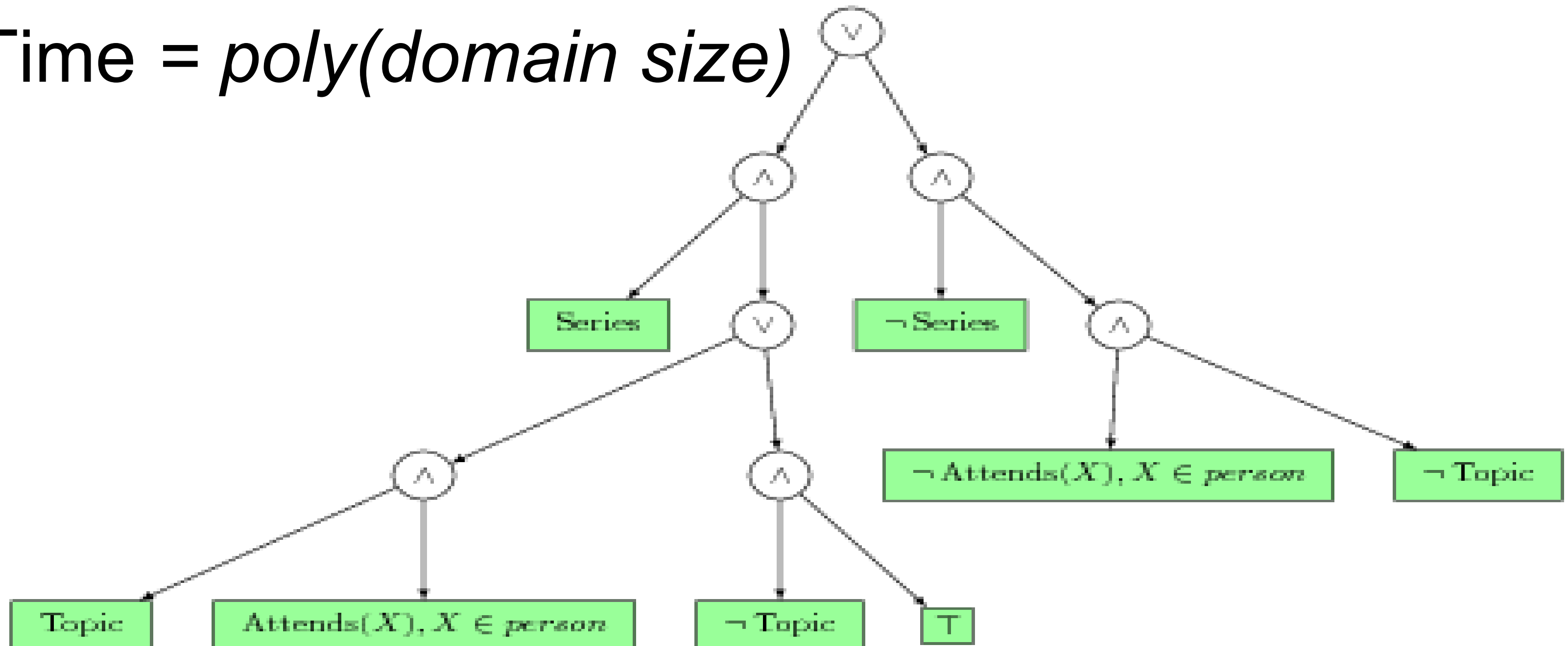
[Poole '03,…]

- Repeatedly apply certain **operators** on the model
  - Lifted multiplication
  - Lifted sum-out
  - …
- Until the desired result is found

Lifted Knowledge Compilation

[Van den Broeck et al '11,…]

- Compile the model into a **"lifted" circuit** ("FO d-DNNF")
  - How? Compilation rules
- Inference = traversing the circuit
  - Time = *poly(domain size)*



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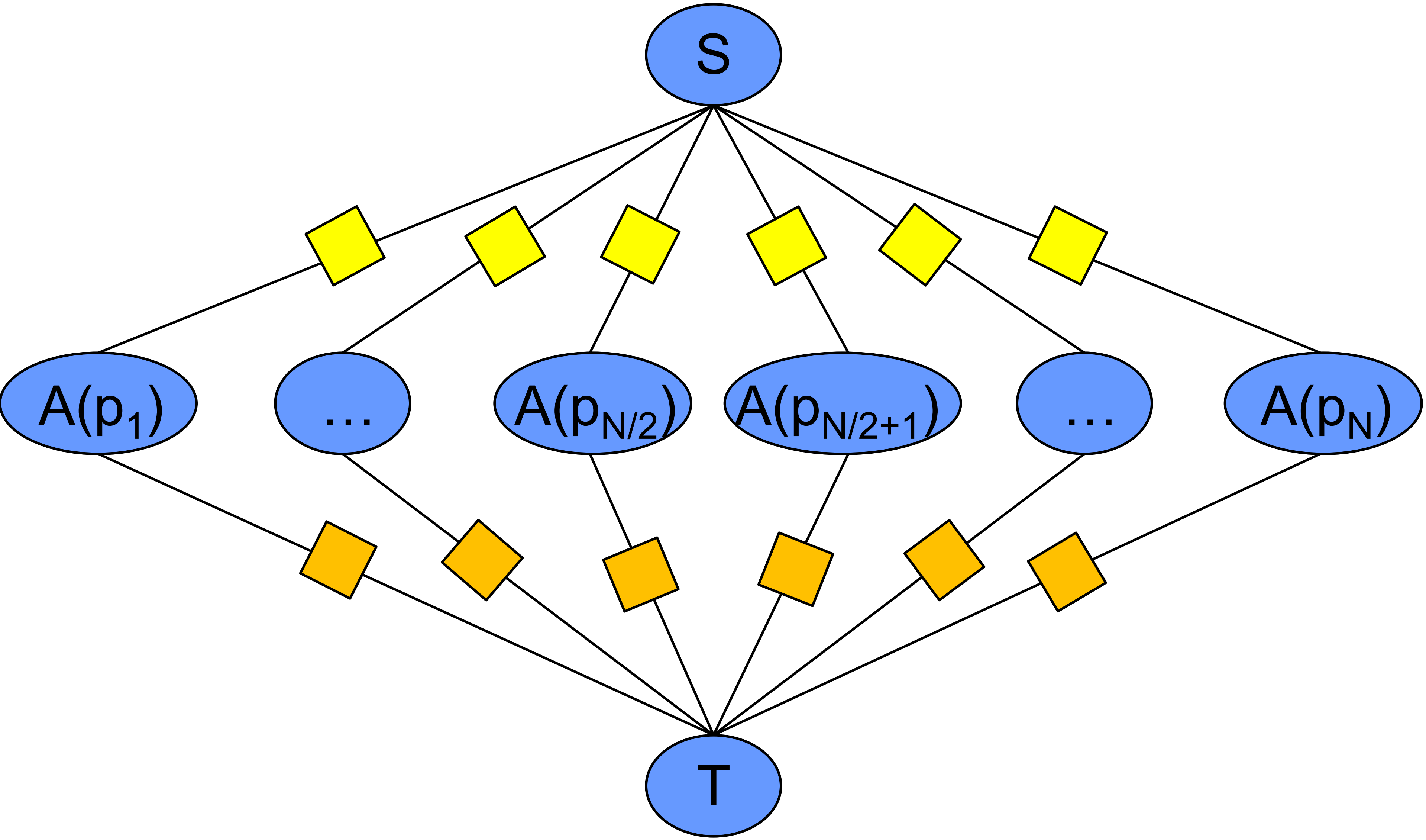
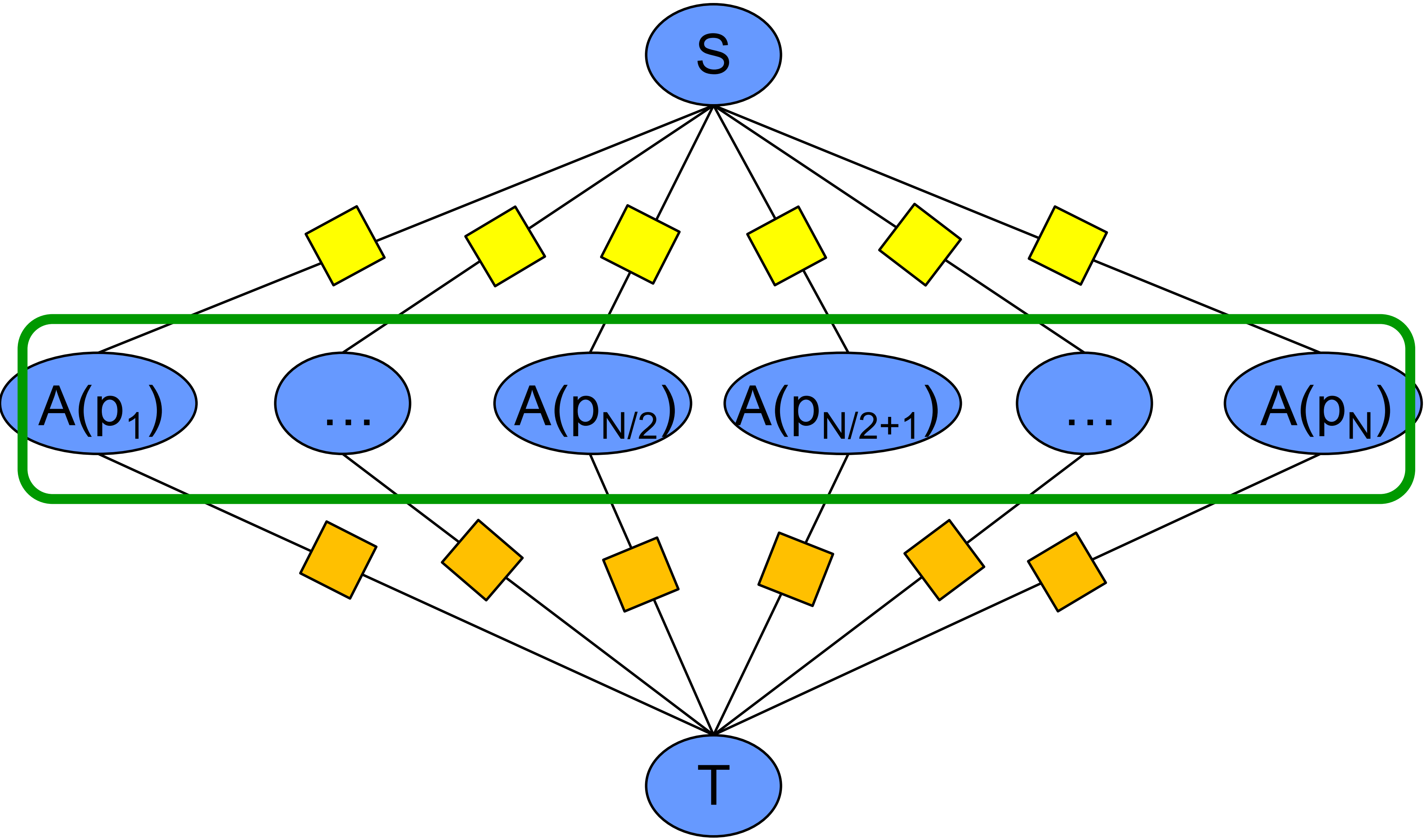
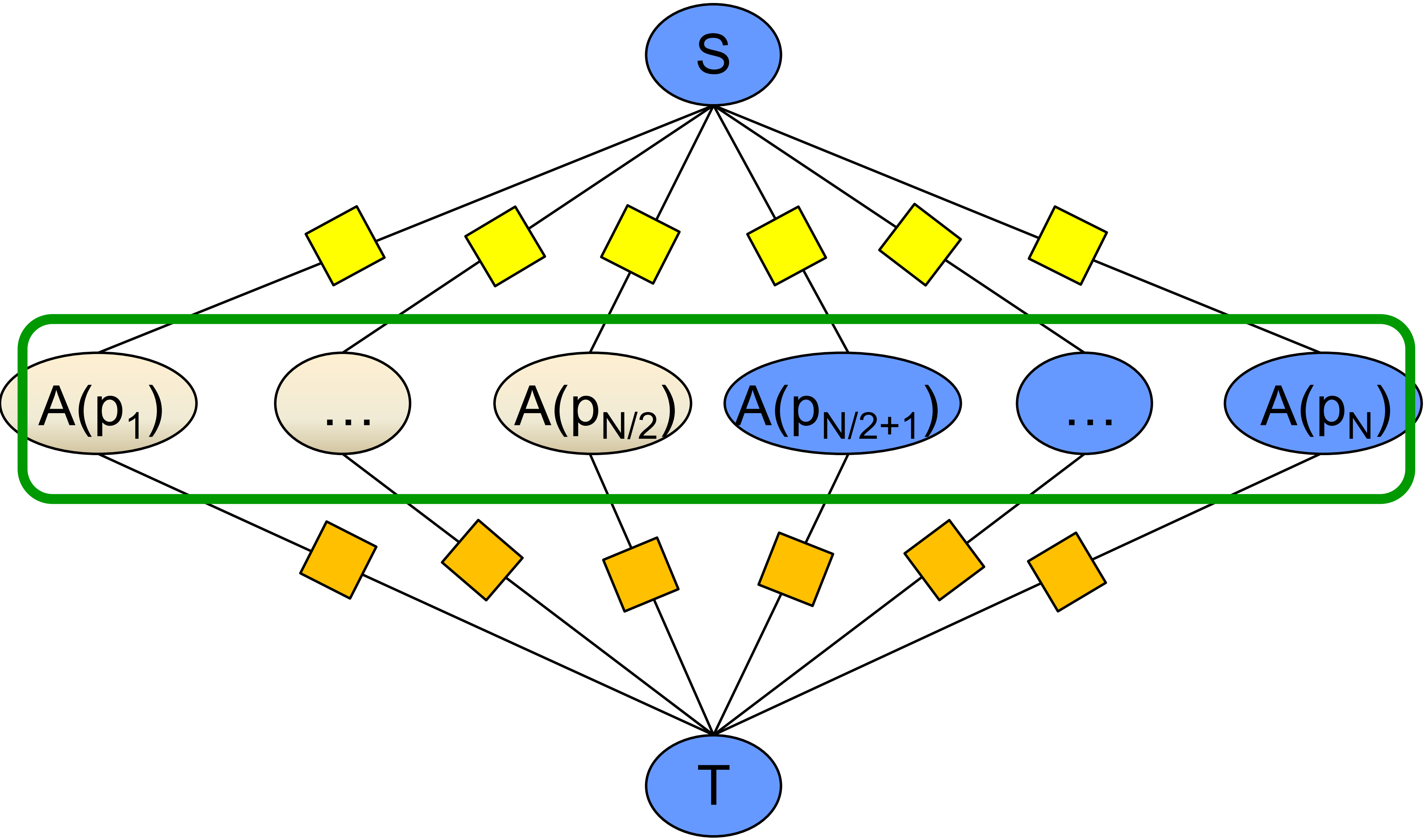

S
A(p1)
…
A(pN/2)
A(pN/2+1)
…
A(pN)
T


S
A(p_1)
...
A($p_{N/2}$)
A($p_{N/2+1}$)
...
A(p_N)
T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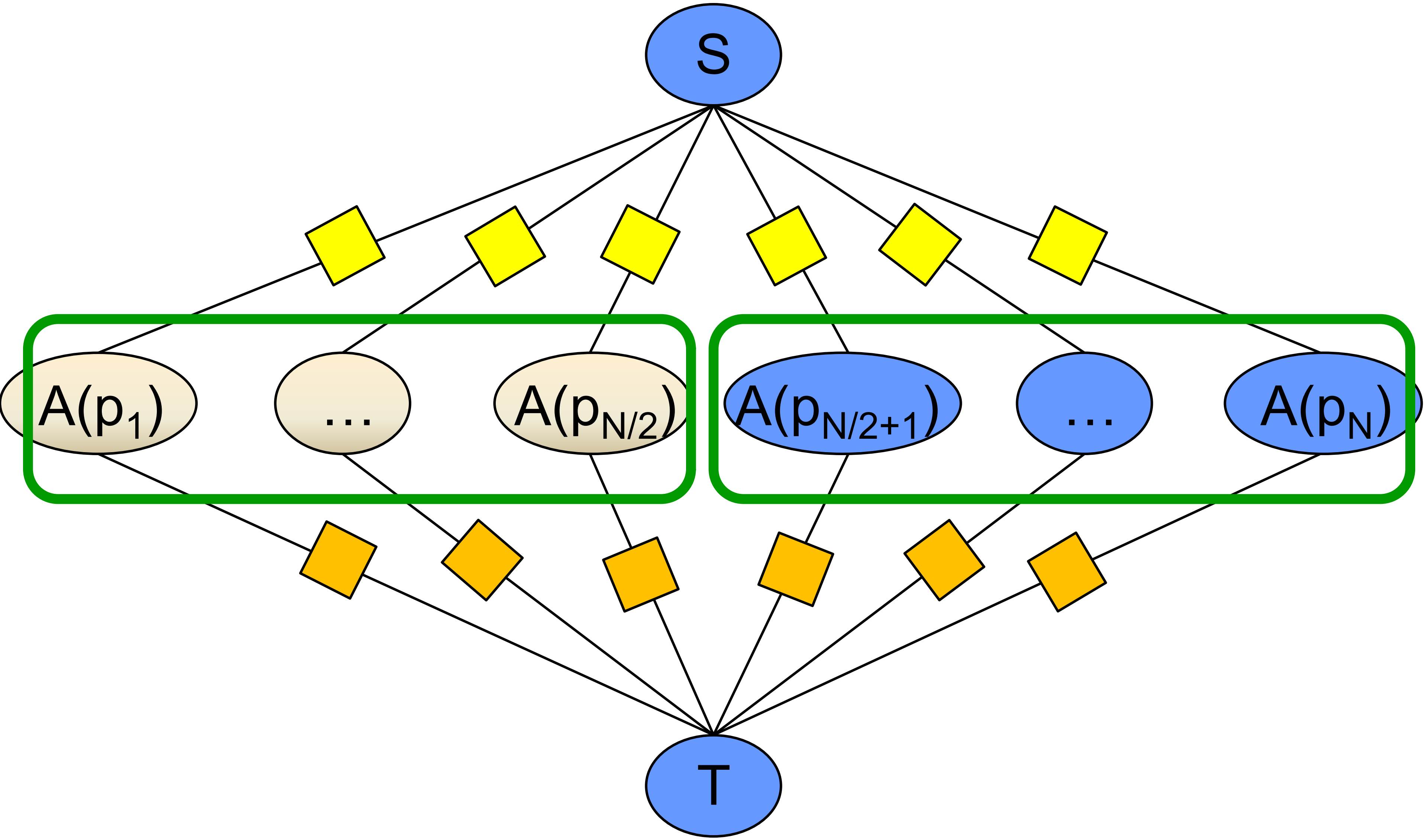

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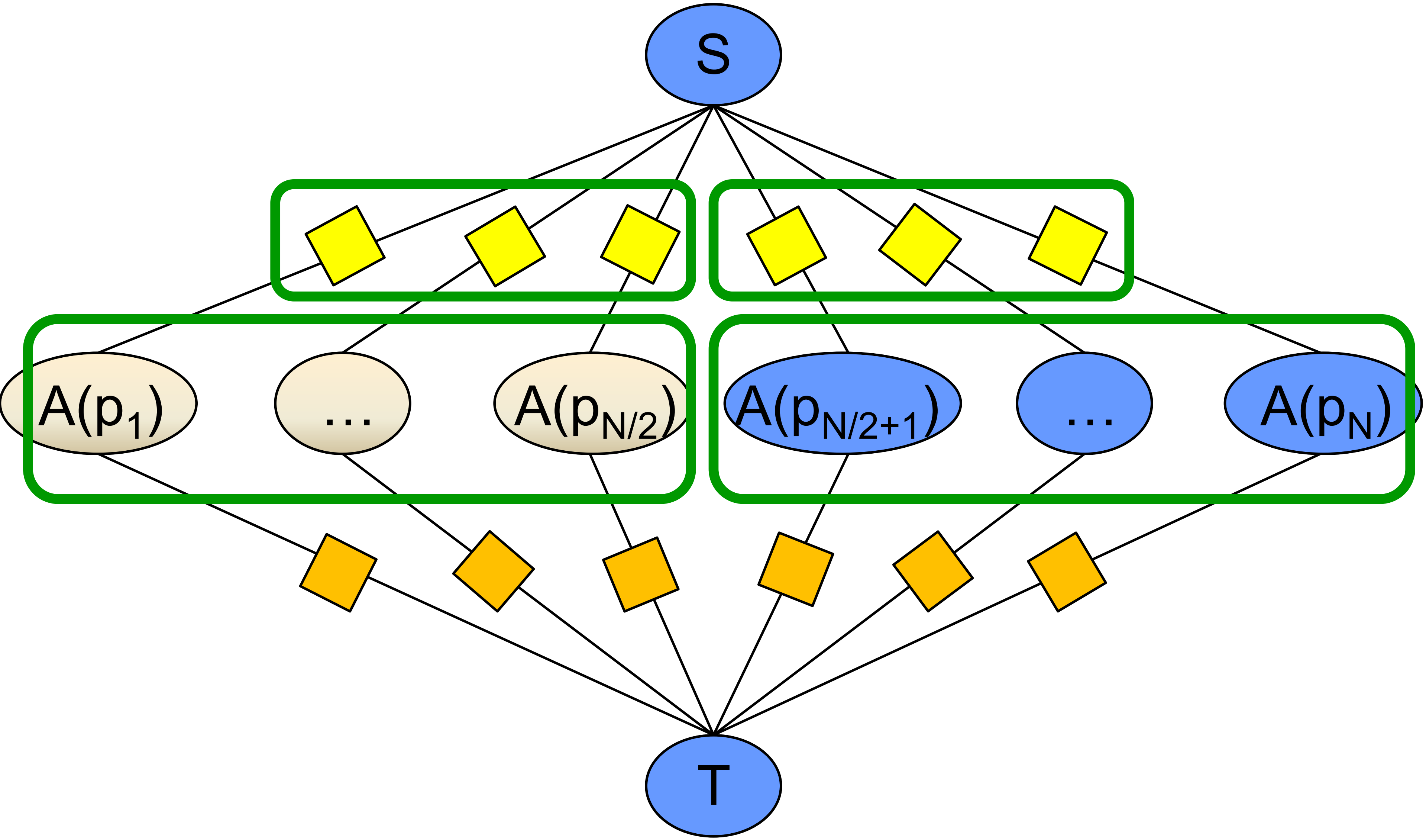

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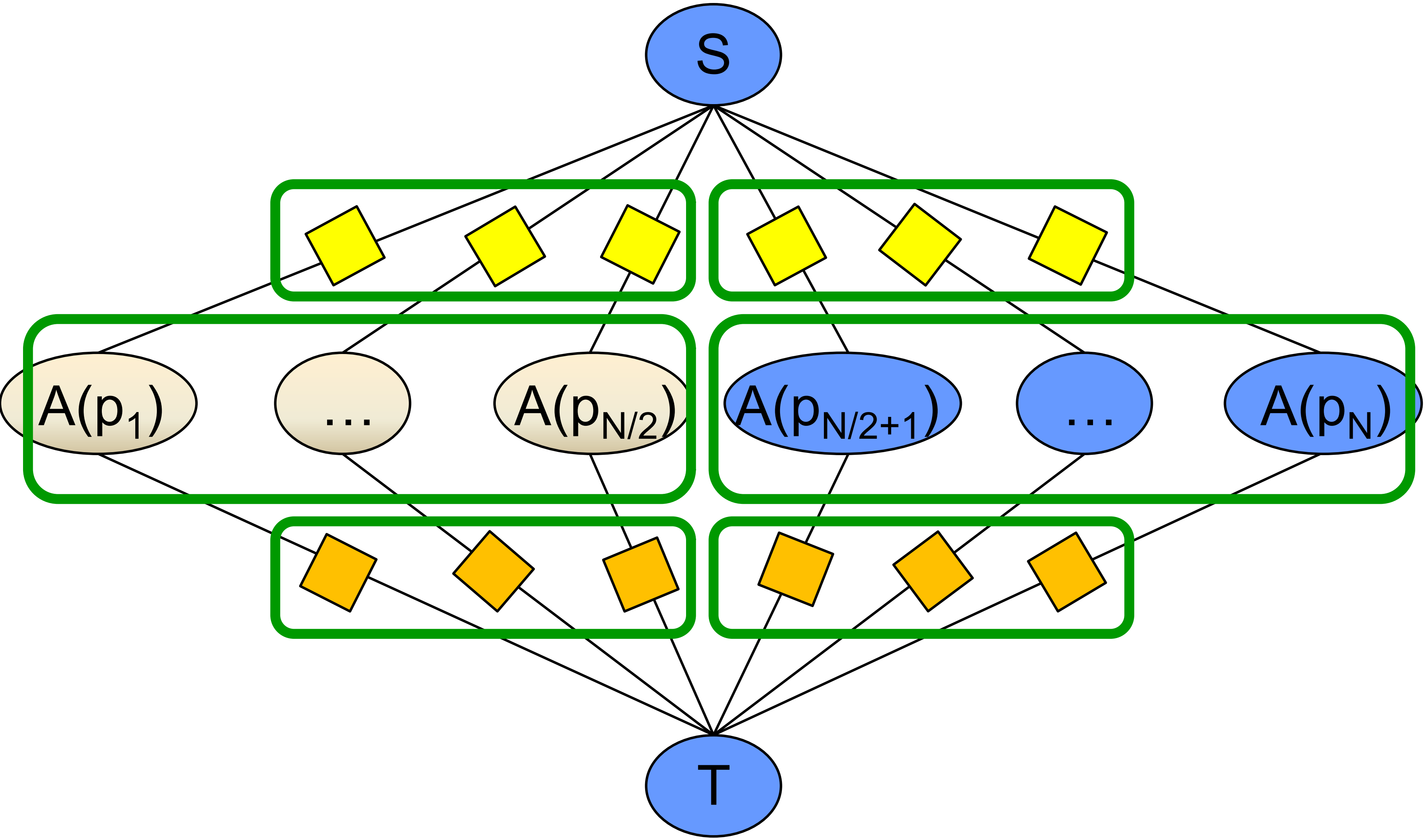
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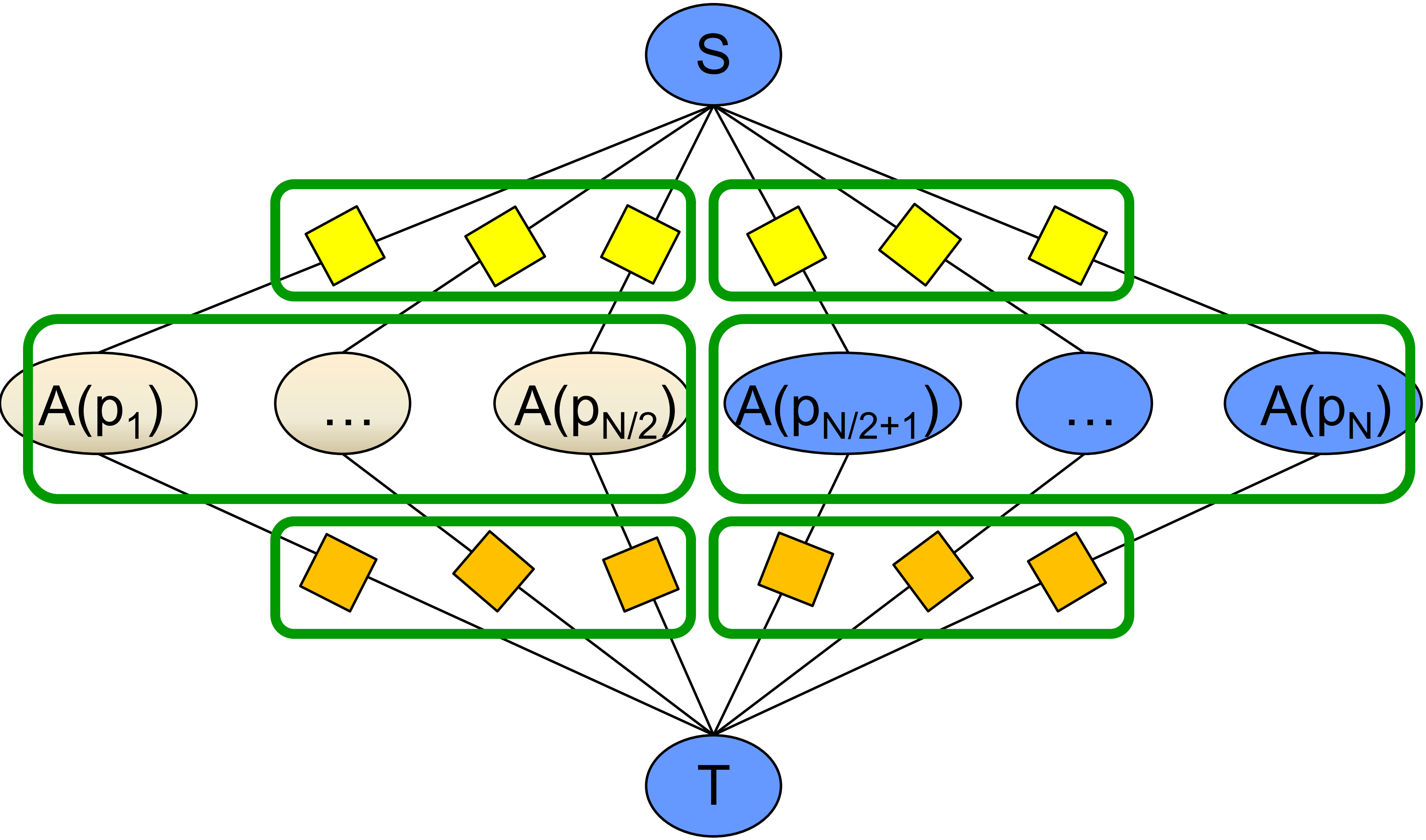
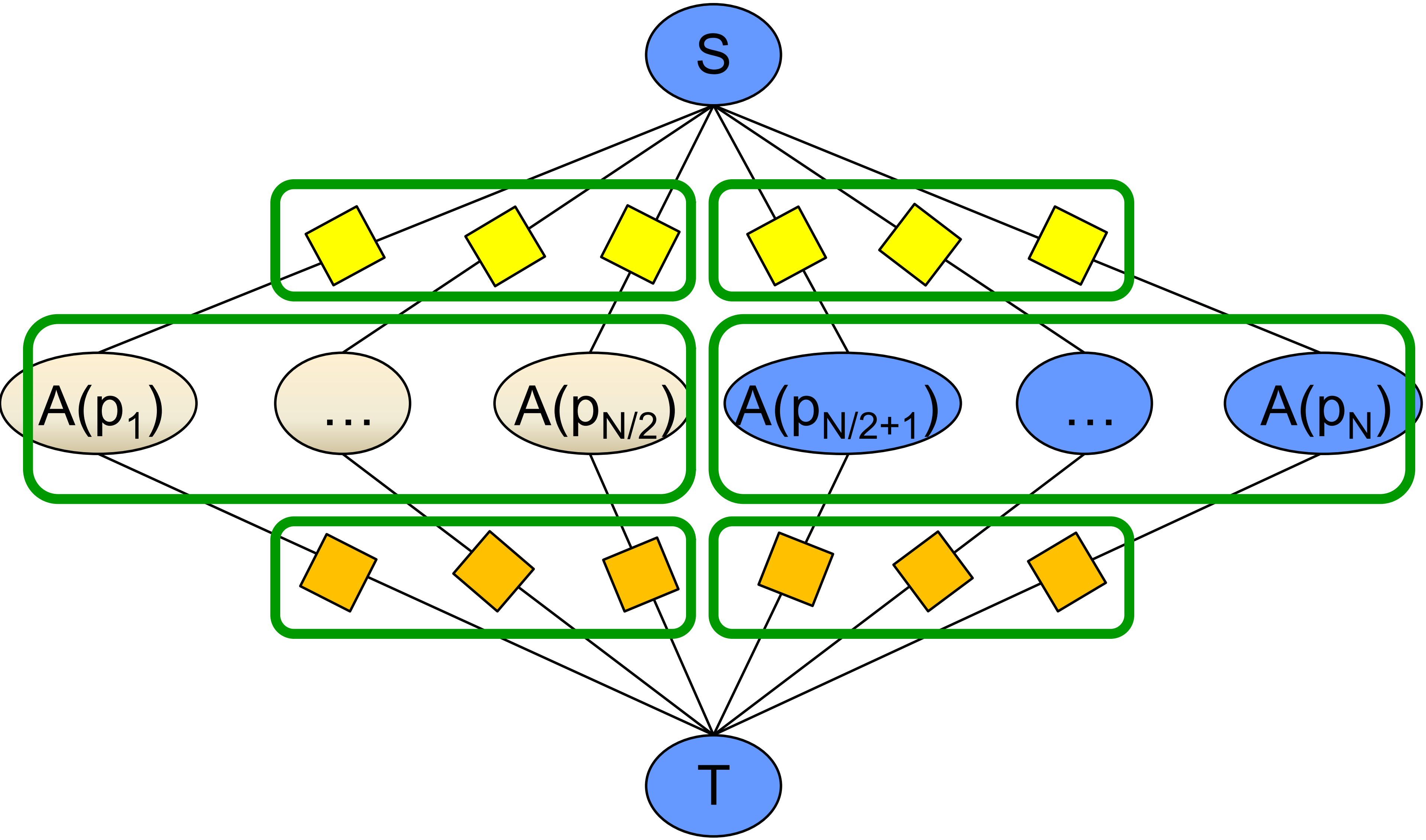


Bigger groups = more lifting !



Bigger groups = more lifting !

The groups are specified by **constraints**

Importance of constraints

[Taghipour et al, AISTATS'12]

- Exact lifted algorithms use a particular constraint language

  group → constraint → can it be expressed in the language ?

- Often leads to unnecessarily small groups → less lifting

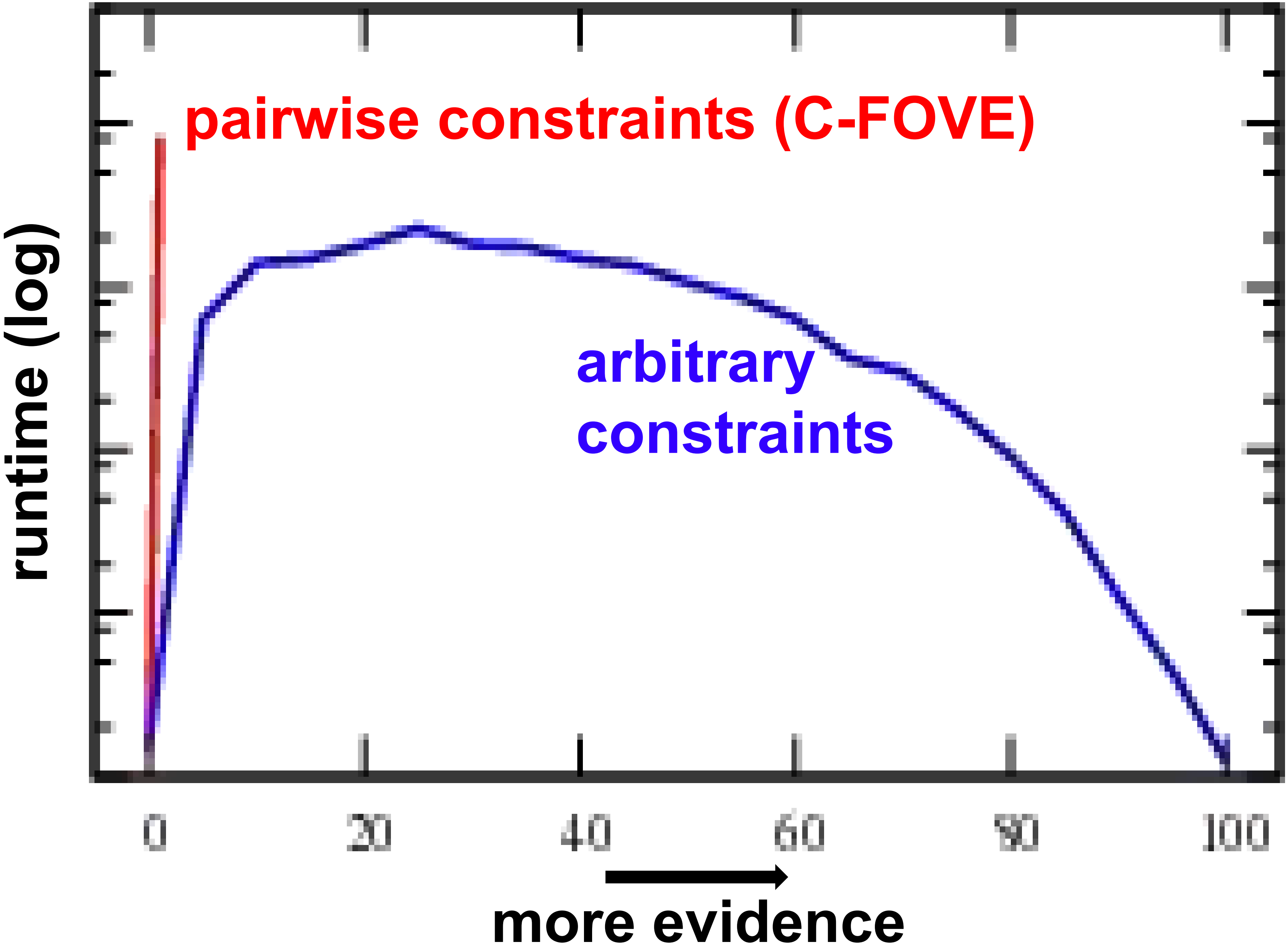
Importance of constraints

[Taghipour et al, AISTATS'12]

- Exact lifted algorithms use a particular constraint language

group → constraint → can it be expressed in the language ?

- Often leads to unnecessarily small groups → less lifting
- We avoid using a particular constraint language
  Instead: **arbitrary constraints**
  + relational algebra


pairwise constraints (C-FOVE)
arbitrary
constraints
runtime (log)
0
20
40
60
80
100
more evidence

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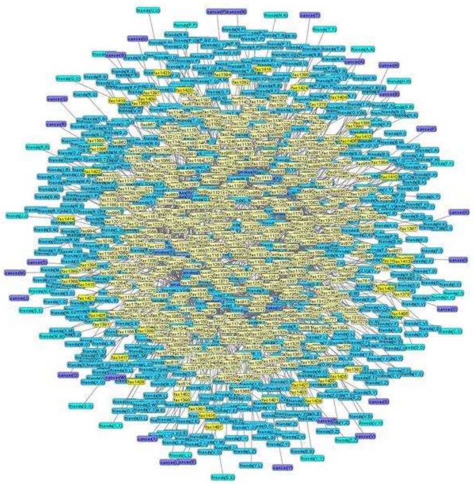
What is Lifted Inference?



- Propositional inference is intractable

  Solution: lifted inference

  “Exploit symmetries”

  “Reason at first-order level”

  “Reason about groups of objects as a whole”

  “Avoid repeated computations”

  “Mimic resolution in theorem proving”

- There is a common understanding but no formal definition of lifted inference!

What is Lifted Inference?

- What is commonly understood as exact lifted inference?

Definition: Domain-Lifted Inference
Complexity of computing P(q|e) in model m is polynomial time in the domain sizes of the logical variables in q,e,m

1.5	Attends(person) → Series
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[Van den Broeck NIPS11]

What is Lifted Inference?

- What is commonly understood as exact lifted inference?

  Definition: Domain-Lifted Inference
  Complexity of computing P(*q*|*e*) in model *m* is polynomial time in the domain sizes of the logical variables in *q*,*e*,*m*

- Possibly exponential in the size of *q*,*e*,*m*

  # predicates, # parfactors, # atoms,
  # arguments, # formulas, # constants in model

[Van den Broeck NIPS11]

What is Lifted Inference?

- Motivation: Large domains lead to intractable propositional inference.
- A formal framework for lifted inference
  - Definition + complexity considerations
  - ~ PAC-learnability (Valiant)
- Other notions, e.g., for approximate inference.

[Van den Broeck NIPS11]

Completeness

- A procedure that is domain-lifted for all models in a class *M* is called complete for *M*

All models in M are "liftable"

- There was no completeness result for existing algorithms

If you give me a model,
I cannot say if grounding will be needed,
untill I run the inference algorithm itself.

[Van den Broeck NIPS11]

Completeness Result

Probabilistic inference in models with

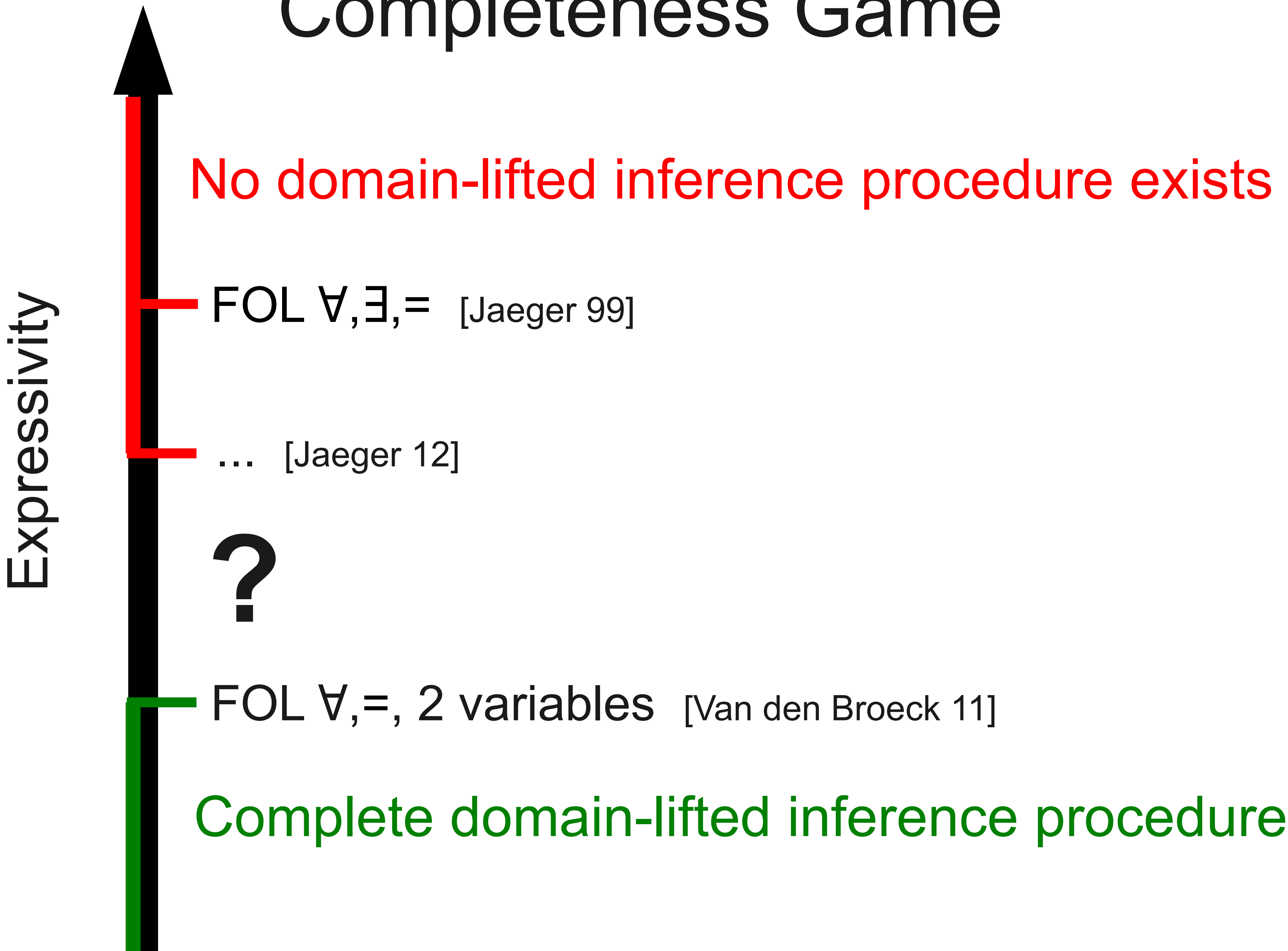
- **universal quantifiers ∀ and**
- **2 logical variables per formula**

is domain-liftable.

- A non-trivial class of models
- First completeness results in exact lifted inference
  - Lifted knowledge compilation procedure
  - Lifted variable elimination procedure

[Van den Broeck NIPS11], [Taghipour et al.]

Completeness Game



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Conditioning

- Task: Probability of query *q* given evidence *e*: P(*q*|*e*)

  Domain-lifted inference is exponential in the size of *e*.

- Can we compute conditional probabilities efficiently?

  Depends on the arity of literals conditioned on:

Literal Arity	Complexity of Conditioning
0	Polynomial
1	Polynomial if supported by compilation
≥ 2	#P-hard

- Positive and negative result for lifted inference

[Van den Broeck, Davis AAAI12]

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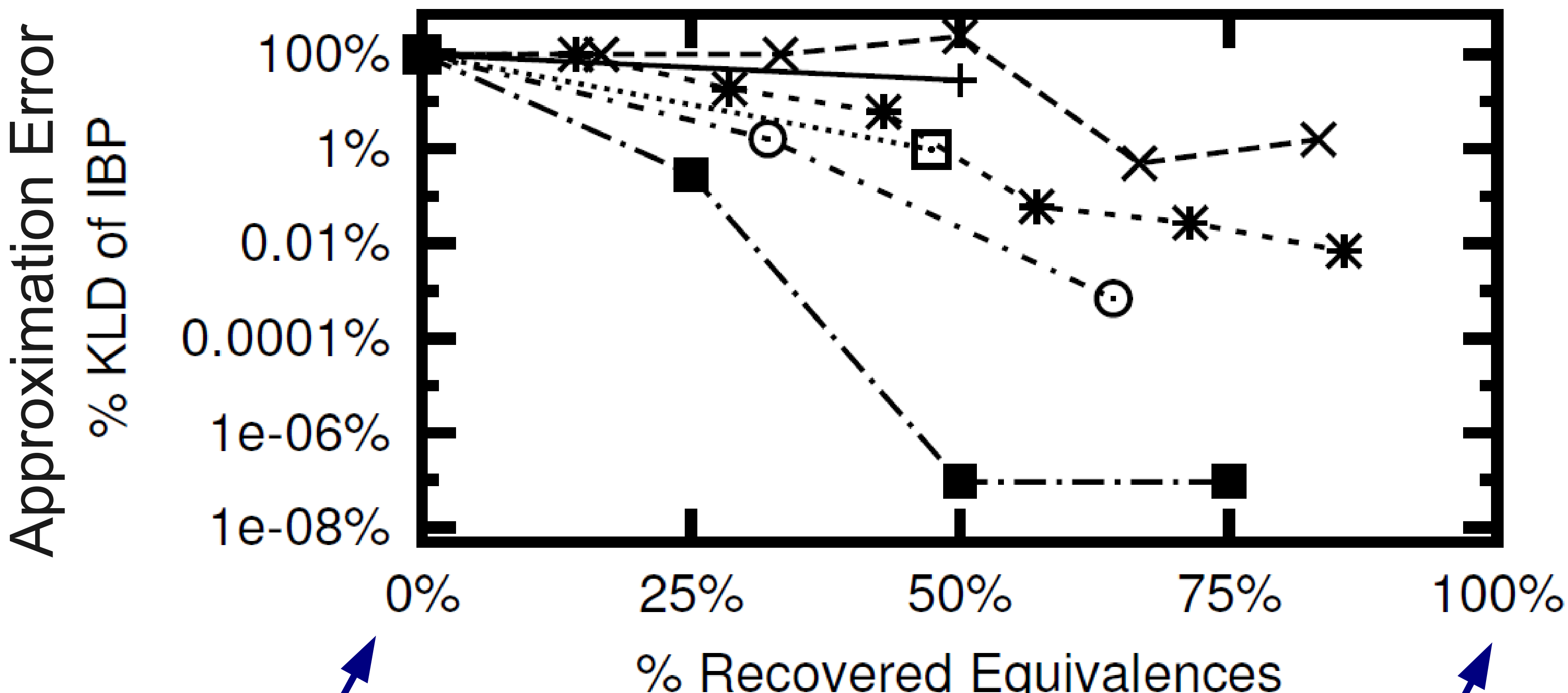
Lifted RCR

- Practical usefulness of lifted inference shown for approximate inference with lifted BP
- Lifted Relax, Compensate and Recover
  (1) Clone all atoms in a model
  (2) Relax equivalences between clones
  (3) Compensate for removed equivalences
  (4) Recover equivalences until model too complex
- Exact lifted inference black box in (3)

[Van den Broeck, Choi, Darwiche]

Lifted RCR



Special case: Lifted BP
Tractable

Exact lifted inference
Intractable

[Van den Broeck, Choi, Darwiche]

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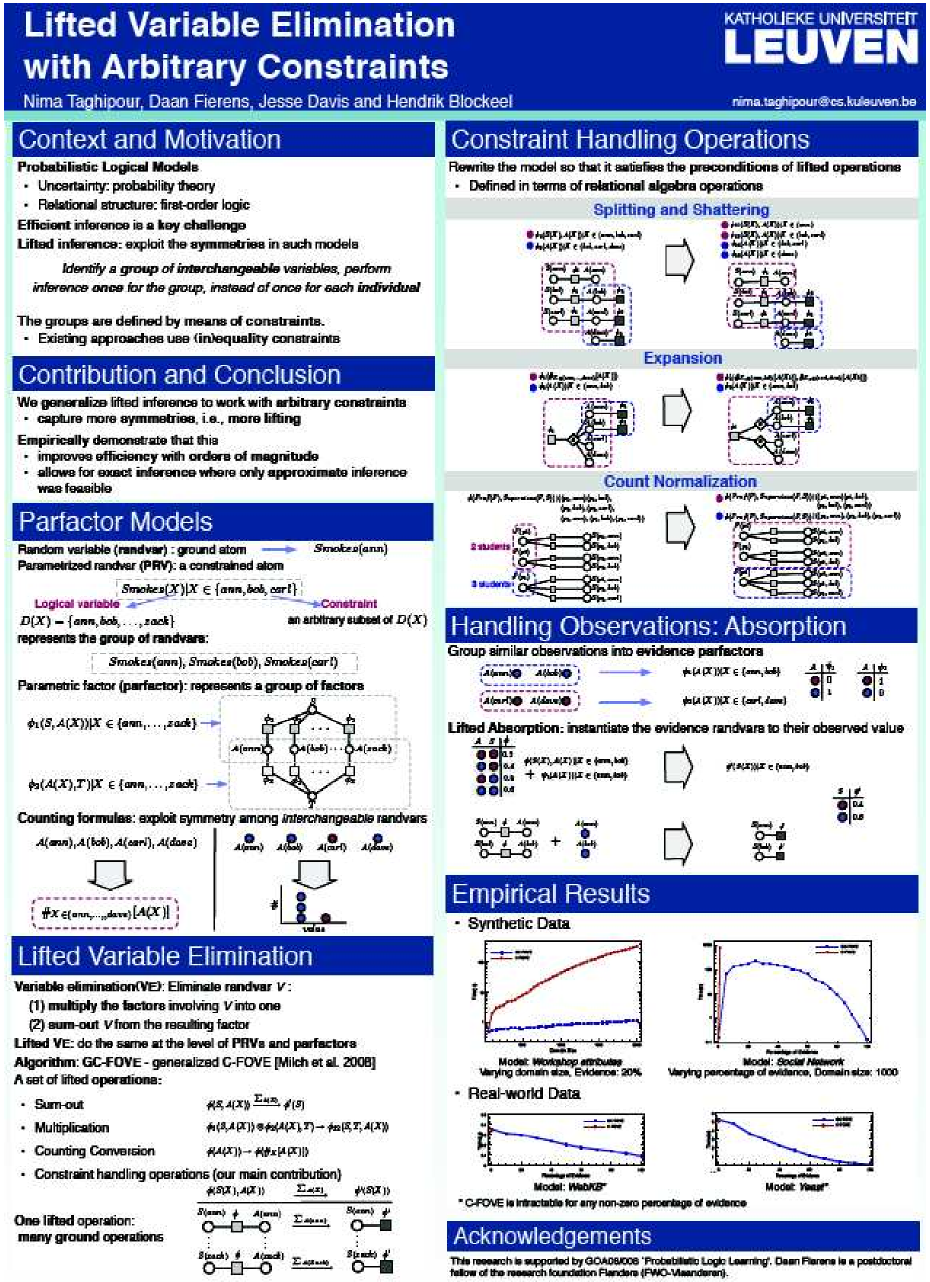
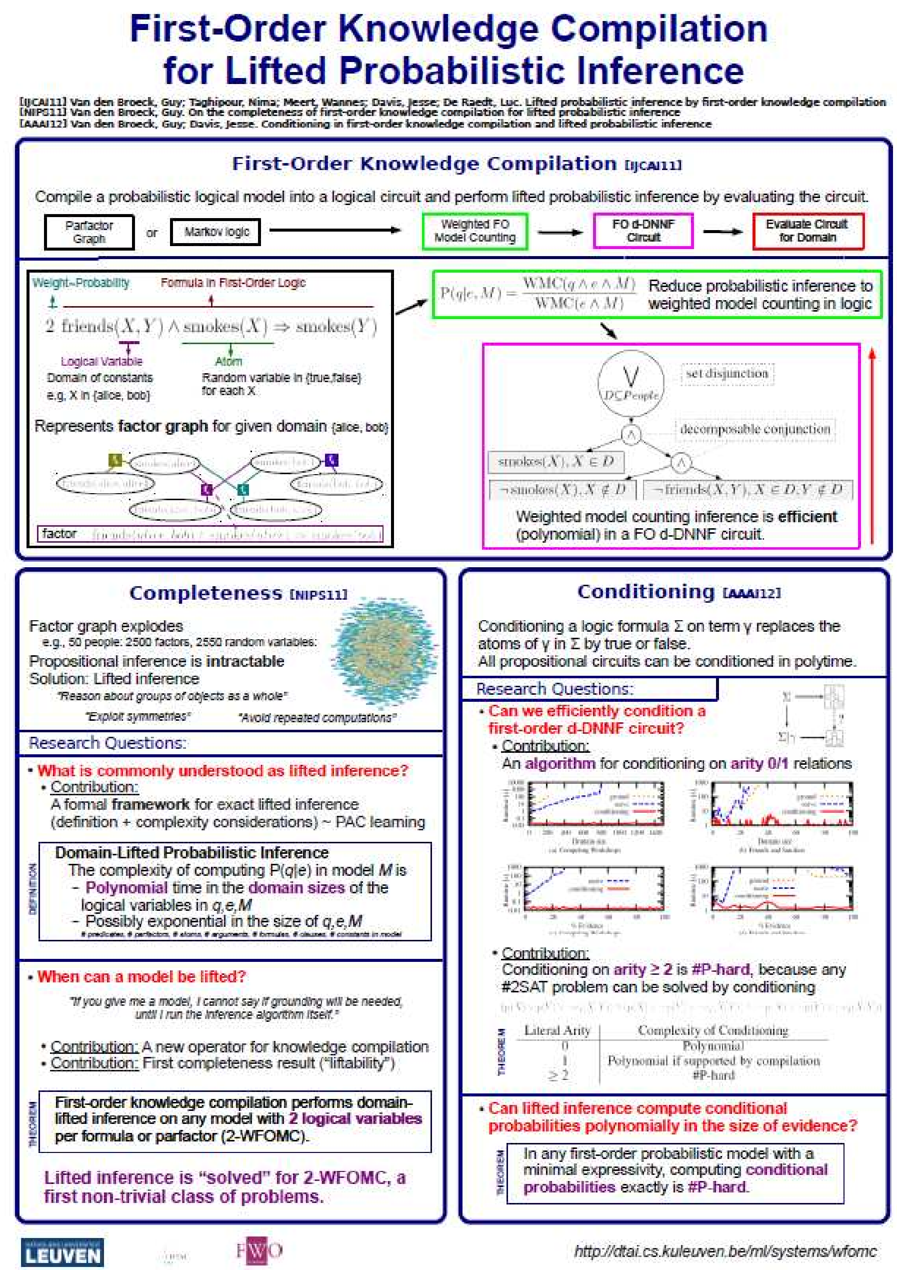
Posters!





Website & Implementation: http://dtai.cs.kuleuven.be/ml/systems/wfomc